

Lyra's Cosmology of Massive String in Anisotropic Bianchi-II Space-time

Anil Kumar Yadav[†] and Abdul Haque[‡]

[†]Department of Physics, Anand Engineering College, Keetham, Agra-282 007, India

E-mail: abanilyadav@yahoo.co.in

[‡]Department of Mathematics, P. G. College, Ghazipur - 233 001, India

E-mail: abdulg_haque@rediffmail.com

Abstract

The paper deals with a spatially homogeneous and totally anisotropic Bianchi II cosmological models representing massive strings in normal gauge for Lyra's manifold. The modified Einstein's field equations have been solved by applying variation law for Hubble's parameter. This law generates two type of solutions for average scale factor, one is of power law type and other is of exponential law type. The power law describes the dynamics of Universe from big bang to present epoch while exponential law seems reasonable to project dynamics of future Universe. It has been found that the displacement actor (β) is a decreasing function of time and it approaches to small positive value at late time, which is collaborated with Halford (1970) as well as recent observations of SN Ia. The study reveals that massive strings dominate in early Universe and eventually disappear from Universe for sufficiently large time, which is in agreement with the current astronomical observations.

Keywords: Massive string, Bianchi-II model, Lyra's manifold, Accelerating universe

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1 Introduction

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of early Universe. It is generally assumed that after the big bang, the Universe may have undergone a series of phase transitions as its temperature lowered down below some critical temperature as predicted by grand unified theories [1]–[6]. At the very early stages of evolution of the Universe, it is believed that during phase transition the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings and monopoles. In particular, cosmic strings are produced in the breaking of U(1) symmetry, are good candidates to seed the formation of galaxies. These cosmic strings have stress-energy, and couple to the gravitational field. Therefore, it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [7] who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier [8] first used this idea in obtaining cosmological solutions in Bianchi-I and Kantowski-Sachs space-times. Stachel [9] has studied massive string.

The observed Universe is satisfactorily described by homogeneous and isotropic models given by the FRW space-time. But at smaller scales, the Universe is neither homogeneous and isotropic nor do we expect the Universe in its early stages to have these properties. The anomalies found in the cosmic microwave background (CMB) and large scale structure observations stimulated a growing interest in anisotropic cosmological model of Universe. Here we confine ourselves to models of Bianchi-type II. Bianchi type-II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of Universe. Asseo and Sol [10] emphasized the importance of Bianchi type-II Universe.

Roy and Banerjee [11] have dealt with locally rotationally symmetric (LRS) cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Wang [12] studied the Letelier model in the context of LRS Bianchi type-II space-time. Recently, Pradhan et al. [13, 14] and Amirhashchi and Zainuddin [15] obtained LRS Bianchi type II cosmological models with perfect fluid distribution of matter and string dust respectively. Belinchon [16, 17] studied Bianchi type-II space-time in connection with massive cosmic string and perfect fluid models with time varying constants under the self-similarity approach respectively. Recently, Kumar [18] has investigated string cosmological models in Bianchi type-II space-time.

In last few decades there has been considerable interest in alternative theories of gravitation. The most important among them being scalar-tensor theories proposed by Lyra [19]. Lyra suggested a modification of Riemannian geometry which may also be considered as a modification of Weyl's geometry. In Lyra's geometry, Weyl's concept of gauge, which is essentially a metrical concept, is modified by the introduction of a gauge function into the structure less manifold. This alternating theory is of interest because it produces effects similar to those produced in Einstein's theory. Also vector field in this theory plays similar role to cosmological constant in general relativity [20, 21]. Several authors Sen and Vanstone [22], Bhamra [23], Singh and Singh [24], Rahaman et al [25, 26], Pradhan et al [27]–[31], Yadav [32] and Yadav et al [33, 34] have studied cosmological models based on Lyra's manifold in various physical contexts.

Recently, Agwarwal, Pandey and Pradhan [35] have studied Bianchi II string cosmological model in Lyra's manifold. They have investigated time varying vector field by assuming direction of string along x-axis. In this paper, we have assumed the direction of string along z-axis, collaborated with Saha [36]. The paper is organized as follows. The metric and the field equations are presented in Section 2. Section 3 deals with exact solutions of the field equations with strings fluid. Physical behavior of the derived models are elaborated in detail. Finally, in Section 4, concluding remarks are given.

2 The metric and field equations

We consider totally anisotropic Bianchi type-II line element, given by

$$ds^2 = -dt^2 + A^2(dx - zdy)^2 + B^2dy^2 + C^2dz^2, \quad (1)$$

where the metric potentials A , B and C are functions of t alone. This ensures that the model is spatially homogeneous.

The field equations (in gravitational units $c = 1, 8\pi G = 1$), in normal gauge for Lyra's manifold, obtained by Sen [37] as

$$R_i^j - \frac{1}{2}g_i^j R + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}g_i^j\phi_k\phi^k = -T_j^i, \quad (2)$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = (0, 0, 0, \beta(t)) \quad (3)$$

and the other symbols have their usual meaning as in Riemannian geometry.

Einstein tensor. The energy-momentum tensor T_j^i for a cloud of massive strings and perfect fluid distribution is taken as

$$T_j^i = (\rho + p)v^iv_j + pg_j^i - \lambda x^ix_j, \quad (4)$$

where p is the isotropic pressure; ρ is the proper energy density for a cloud strings with particles attached to them; λ is the string tension density; $v^i = (0, 0, 0, 1)$ is the four-velocity of the particles, and x^i is a unit space-like vector representing the direction of string. The vectors v^i and x^i satisfy the conditions

$$v_iv^i = -x_ix^i = -1, \quad v^ix_i = 0. \quad (5)$$

Choosing x^i parallel to $\partial/\partial z$, we have

$$x^i = (0, 0, C^{-1}, 0). \quad (6)$$

Here the cosmic string has been directed along z-direction in order to satisfy the condition $T_1^1 = T_2^2$. As a result, the off-diagonal component of Einstein tensor, viz., $G_2^1 = z(T_2^2 - T_1^1)$ vanishes. A detailed analysis about the choice of energy-momentum tensor for Bianchi type-II models is given by Saha [36].

If the particle density of the configuration is denoted by ρ_p , then

$$\rho = \rho_p + \lambda. \quad (7)$$

The modified Einstein's field equations (2) for line element (1) with energy-momentum tensor (4), lead to the following set of independent differential equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{3}{4}\frac{A^2}{B^2C^2} - \frac{3}{4}\beta^2 = -p, \quad (8)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{1}{4} \frac{A^2}{B^2 C^2} - \frac{3}{4} \beta^2 = -p, \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4} \frac{A^2}{B^2 C^2} - \frac{3}{4} \beta^2 = -p + \lambda, \quad (10)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{4} \frac{A^2}{B^2 C^2} + \frac{3}{4} \beta^2 = \rho. \quad (11)$$

Here, and in what follows, an over dot indicates ordinary differentiation with respect to t . The energy conservation equation $T^{ij}_{;j} = 0$, leads to the following expression:

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) - \lambda \frac{\dot{C}}{C} = 0, \quad (12)$$

and conservation of R. H. S. of eq. (2) leads to

$$\left(R_i^j - \frac{1}{2} g_i^j R \right)_{;j} + \frac{3}{2} (\phi_i \phi^j)_{;j} - \frac{3}{4} (g_i^j \phi_k \phi^k)_{;j} = 0 \quad (13)$$

Equation (13) reduces to

$$\frac{3}{2} \phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma_{lj}^j \right] + \frac{3}{2} \left[\frac{\partial \phi_i}{\partial x^j} - \phi_l \Gamma_{ij}^l \right] - \frac{3}{4} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma_{lj}^k \right] - \frac{3}{4} g_i^j \phi^k \left[\frac{\partial \phi_k}{\partial x^j} - \phi_l \Gamma_{kj}^l \right] = 0 \quad (14)$$

Equation (14) is identically satisfied for $i = 1, 2, 3$. for $i = 4$, eq. (14) reduces to

$$\begin{aligned} \frac{3}{2} \beta \left[\frac{\partial (g^{44} \phi_4)}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] + \frac{3}{2} g^{44} \phi_4 \left[\frac{\partial \phi_4}{\partial t} - \phi_4 \Gamma_{44}^4 \right] - \frac{3}{4} g_4^4 \phi_4 \left[\frac{\partial \phi^4}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] \\ - \frac{3}{4} g_4^4 g^{44} \phi^4 \left[\frac{\partial \phi_4}{\partial t} - \phi^4 \Gamma_{44}^4 \right] = 0 \end{aligned} \quad (15)$$

which leads to

$$\frac{3}{2} \beta \dot{\beta} + \frac{3}{2} \beta^2 \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (16)$$

Thus eq. (12) combined with eq. (16) is the resulting equation when energy conservation equation is satisfied in the given system.

3 Solutions of the Field Equations

Equations (8)-(11) and (16) are five equations in seven unknown parameters A , B , C , p , ρ , λ and β . Two additional constraints relating these parameters are required to obtain explicit solutions of the system.

First, we utilize the special law of variation for the Hubble's parameter given by Berman [38], which yields a constant value of deceleration parameter. Here, the law reads as

$$H = \ell (ABC)^{-\frac{n}{3}}, \quad (17)$$

where $\ell > 0$ and $n \geq 0$ are constants. Such type of relations have already been considered by Berman and Gomide [39] for solving FRW models. Later on, many authors (see Kumar and Singh [40], Pradhan et al [41], Yadav [42] and references therein) have studied Bianchi type models by using the special law for Hubble's parameter that yields constant value of deceleration parameter.

Considering $(ABC)^{\frac{1}{3}}$ as the average scale factor of the anisotropic Bianchi-II space-time, the average Hubble's parameter may be written as

$$H = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \quad (18)$$

Equating the right hand sides of (13) and (14), and integrating, we obtain

$$ABC = (n\ell t + c_1)^{\frac{3}{n}}, \quad (n \neq 0) \quad (19)$$

$$ABC = c_2^3 e^{3\ell t}, \quad (n = 0) \quad (20)$$

where c_1 and c_2 are constants of integration. Thus, the law (17) provides power-law (19) and exponential-law (20) of expansion of the Universe.

Following Pradhan and Chouhan [41], we assume that the component σ_1^1 of the shear tensor σ_i^j is proportional to the expansion scalar (θ) . This condition leads to the following relation between the metric potentials:

$$A = (BC)^m, \quad (21)$$

where m is a positive constant.

Now, subtracting (9) from (8), and taking integral of the resulting equation two times, we get

$$\frac{B}{A} = c_4 \exp \left[\int \left\{ \frac{1}{ABC} \int \frac{A^3}{BC} dt \right\} dt + c_3 \int \frac{1}{ABC} dt \right], \quad (22)$$

where c_3 and c_4 are constants of integration. In the following subsections, we discuss the string cosmology using the power-law (19) and exponential-law (20) of expansion of the Universe.

3.1 Lyra's Cosmology of String fluid with Power-law

Solving the equations (16), (12) and (19), we obtain the metric functions as

$$A(t) = (n\ell t + c_1)^{\frac{3m}{n(m+1)}}, \quad (23)$$

$$B(t) = c_4 (n\ell t + c_1)^{\frac{3m}{n(m+1)}} \exp \left[\frac{(m+1)^2}{2\ell^2 M} (n\ell t + c_1)^{\frac{6(m-1)}{n(m+1)}+2} + \frac{c_3}{\ell(n-3)} (n\ell t + c_1)^{\frac{n-3}{n}} \right], \quad (24)$$

$$C(t) = c_4^{-1} (n\ell t + c_1)^{\frac{3(1-m)}{n(m+1)}} \exp \left[-\frac{(m+1)^2}{2\ell^2 M} (n\ell t + c_1)^{\frac{6(m-1)}{n(m+1)}+2} - \frac{c_3}{\ell(n-3)} (n\ell t + c_1)^{\frac{n-3}{n}} \right], \quad (25)$$

where $M = (9m - 3 + mn + n)(3m - 3 + mn + n)$ and $n \neq 3$. Thus, the metric (1) is completely determined. Equation (16) leads to

$$\frac{\dot{\beta}}{\beta} = - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (26)$$

which reduces to

$$\frac{\dot{\beta}}{\beta} = - \frac{3\ell}{(n\ell t + c_1)} \quad (27)$$

Integrating equation (27), we obtain

$$\beta = k_1 (n\ell t + c_1)^{-\frac{3}{n}} \quad (28)$$

where k_1 is the constant of integration.

The expressions for the isotropic pressure (p), the proper energy density (ρ), the string tension (λ) and the particle density (ρ_p) for the above model are obtained as

$$\begin{aligned} p = & \frac{3\ell^2[n(m+1) - 3(m^2 - m + 1)]}{(m+1)^2} (n\ell t + c_1)^{-2} + \frac{3(m+1)(n+1)}{4(9m-3+mn+n)} (n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}} \\ & + \frac{3c_3\ell(1-2m)}{m+1} (n\ell t + c_1)^{-\frac{3}{n}-1} - \frac{(m+1)^2}{\ell^2(9m-3+mn+n)^2} (n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-2]} \\ & - \frac{2c_3(m+1)}{\ell(9m-3+mn+n)} (n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-1]} - c_3^2 (n\ell t + c_1)^{-\frac{6}{n}} - \frac{3}{4} k^2 (n\ell t + c_1)^{-\frac{6}{n}}, \end{aligned} \quad (29)$$

$$\begin{aligned} \rho = & \frac{9\ell^2 m(2-m)}{(m+1)^2} (n\ell t + c_1)^{-2} + \frac{15-33m-mn-n}{4(9m-3+mn+n)} (n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}} \\ & + \frac{3c_3\ell(1-2m)}{m+1} (n\ell t + c_1)^{-\frac{3}{n}-1} - \frac{(m+1)^2}{\ell^2(9m-3+mn+n)^2} (n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-2]} \\ & - \frac{2c_3(m+1)}{\ell(9m-3+mn+n)} (n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-1]} - c_3^2 (n\ell t + c_1)^{-\frac{6}{n}} + \frac{3}{4} k^2 (n\ell t + c_1)^{-\frac{6}{n}}, \end{aligned} \quad (30)$$

$$\lambda = \frac{3\ell^2(2m-1)(3-n)}{m+1}(n\ell t + c_1)^{-2} + 2(n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}}, \quad (31)$$

$$\begin{aligned} \rho_p = & \frac{3\ell^2[3m(2-m) + (3-n)(1-2m)(m+1)]}{(m+1)^2}(n\ell t + c_1)^{-2} + \frac{3(13-35m-3mn-3n)}{4(9m-3+mn+n)}(n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}} \\ & + \frac{3c_3\ell(1-2m)}{m+1}(n\ell t + c_1)^{-\frac{3}{n}-1} - \frac{(m+1)^2}{\ell^2(9m-3+mn+n)^2}(n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-2]} \\ & - \frac{2c_3(m+1)}{\ell(9m-3+mn+n)}(n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-1]} - c_3^2(n\ell t + c_1)^{-\frac{6}{n}} + \frac{3}{4}k^2(n\ell t + c_1)^{-\frac{6}{n}}. \end{aligned} \quad (32)$$

The critical energy density (ρ_c) and density parameter (Ω) are given by

$$\rho_c = 3\ell^2(n\ell t + c_1)^{-2} \quad (33)$$

$$\begin{aligned} \Omega = & \frac{3m(2-m)}{(m+1)^2} + \frac{15-33m-mn-n}{12(9m-3+mn+n)\ell^2}(n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}+2} \\ & + \frac{c_3\ell(1-2m)}{\ell(m+1)}(n\ell t + c_1)^{-\frac{3}{n}+1} - \frac{(m+1)^2}{3\ell^4(9m-3+mn+n)^2}(n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-4]} \\ & - \frac{2c_3(m+1)}{3\ell^3(9m-3+mn+n)}(n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-3]} - \frac{c_3^2}{3\ell^2}(n\ell t + c_1)^{-\frac{6}{n}+2} + \frac{k^2}{4\ell^2}(n\ell t + c_1)^{-\frac{6}{n}+2}, \end{aligned} \quad (34)$$

The average Hubble's parameter, expansion scalar, shear scalar spatial volume (V) and anisotropy parameter (\bar{A}) spatial volume (V) and anisotropy parameter (\bar{A}) of the model are, respectively given by

$$H = \frac{1}{3}(H_x + H_y + H_z) = \ell(n\ell t + c_1)^{-1}, \quad (35)$$

$$\theta = 3H = 3\ell(n\ell t + c_1)^{-1}, \quad (36)$$

$$\begin{aligned} \sigma^2 = & \frac{1}{6} [(H_x - H_y)^2 + (H_y - H_z)^2 + (H_z - H_x)^2] \\ = & \frac{3\ell^2(2m-1)^2}{(m+1)^2}(n\ell t + c_1)^{-2} - \frac{15-33m-mn-n}{4(9m-3+mn+n)}(n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}} \\ & - \frac{3c_3\ell(1-2m)}{m+1}(n\ell t + c_1)^{-\frac{3}{n}-1} + \frac{(m+1)^2}{\ell^2(9m-3+mn+n)^2}(n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-2]} \\ & + \frac{2c_3(m+1)}{\ell(9m-3+mn+n)}(n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-1]} + c_3^2(n\ell t + c_1)^{-\frac{6}{n}}. \end{aligned} \quad (37)$$

$$V = ABC = (n\ell t + c_1)^{\frac{3}{n}}, \quad (38)$$

$$\begin{aligned} \bar{A} = & \frac{2\sigma^2}{3H^2} = \frac{2(2m-1)^2}{(m+1)^2} - \frac{15-33m-mn-n}{6\ell^2(9m-3+mn+n)}(n\ell t + c_1)^{-\frac{6(1-m)}{n(m+1)}+2} \\ & - \frac{2c_3(1-2m)}{\ell(m+1)}(n\ell t + c_1)^{-\frac{3}{n}+1} + \frac{2(m+1)^2}{3\ell^4(9m-3+mn+n)^2}(n\ell t + c_1)^{-[\frac{12(1-m)}{n(m+1)}-4]} \\ & + \frac{4c_3(m+1)}{3\ell^3(9m-3+mn+n)}(n\ell t + c_1)^{-[\frac{3(3-m)}{n(m+1)}-3]} + \frac{2c_3^2}{3\ell^2}(n\ell t + c_1)^{-\frac{6}{n}+2}. \end{aligned} \quad (39)$$

We observe that all the parameters diverge at $t = -c_1/n\ell$. Therefore, the model has a singularity at $t = -c_1/n\ell$, which can be shifted to $t = 0$ by choosing $c_1 = 0$. This singularity is of Point Type as all the scale factors vanish at $t = -c_1/n\ell$. The cosmological evolution of Bianchi-II space-time is expansionary since all the scale factors monotonically increase with time (see, **Fig.1**). So, the Universe starts expanding with a big bang singularity in the derived model. The parameters p , ρ , ρ_p and λ start off with extremely large values. In particular, the large values of ρ_p and λ in the beginning suggest that strings dominate the early Universe.

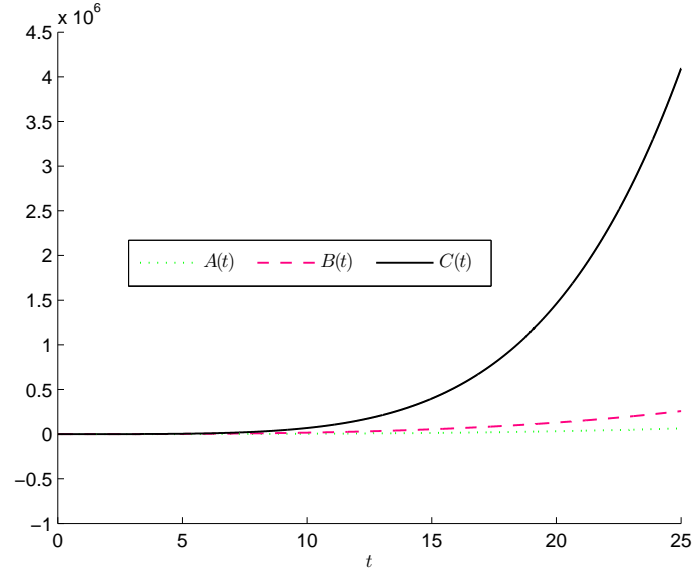


Figure 1: Scale factors versus time (t).

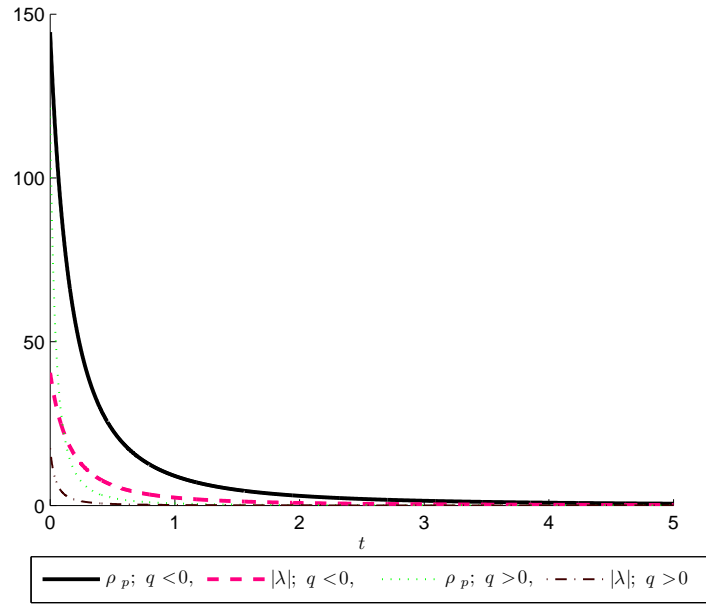


Figure 2: Particle energy density (ρ_p) and string tension density ($|\lambda|$) versus time (t).

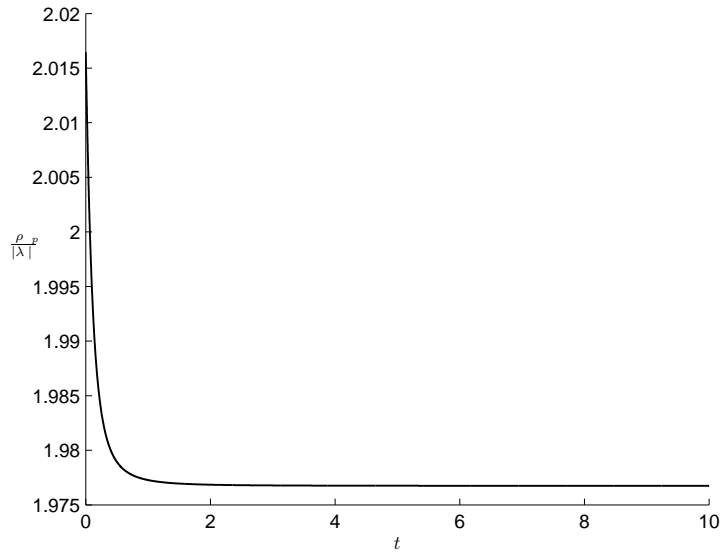


Figure 3: Plot of $\frac{\rho_p}{|\lambda|^p}$ versus time (t).

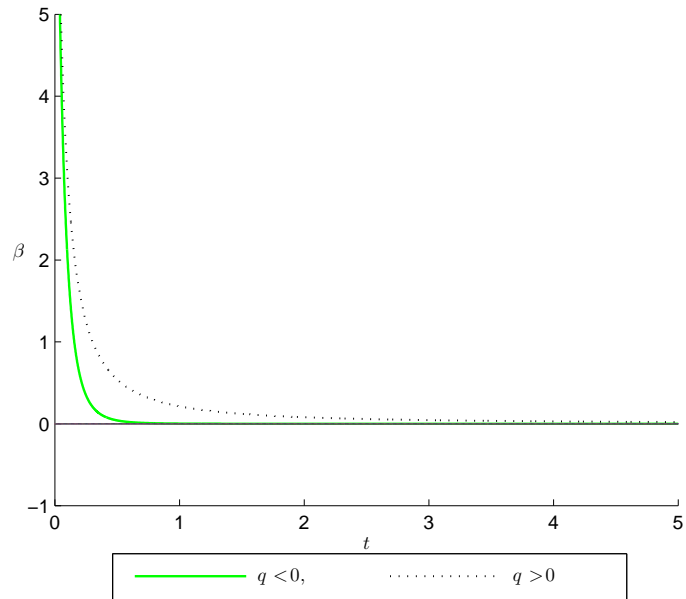


Figure 4: Plot of displacement vector (β) versus time (t).

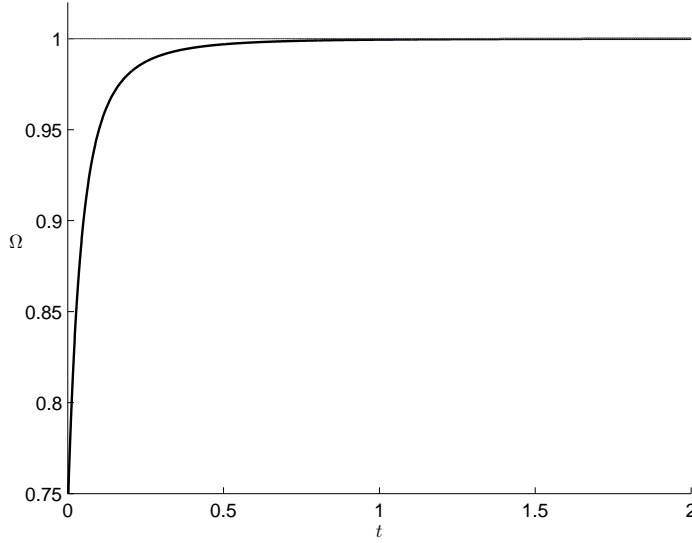


Figure 5: Plot of density parameter (Ω) versus time (t).

For sufficiently large times, ρ_p and λ become negligible. Therefore, the strings disappear from the Universe for larger times.

From **Fig. 1**, it is observed that during cosmic expansion, $C(t) > B(t) > A(t)$. Thus the derived model can be utilised to describe the anisotropy of early Universe. In particular for $m = 0.5$, at late time, the directional scale factors vary as

$$A(t) \approx (nlt + c_1)^{\frac{1}{n}}, \quad B(t) \approx (nlt + c_1)^{\frac{1}{n}}, \quad C(t) \approx (nlt + c_1)^{\frac{1}{n}}.$$

Therefore, isotropy is achieved in the derived model for $m = 0.5$.

The value of DP (q) is found to be

$$q = -1 + \frac{\dot{H}}{H^2} = n - 1, \quad (40)$$

which is a constant. A positive sign of q , i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q , i.e., $0 < n < 1$ indicates acceleration. The expansion of the Universe at a constant rate corresponds to $n = 1$, i.e., $q = 0$. Also, recent observations of SN Ia [43]-[47] reveal that the present Universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$. It follows that in the derived model, one can choose the values of DP consistent with the observations. For $n < 1$, the model is accelerating whereas for $n > 1$ it goes to decelerating phase. In what follows, we compare the two modes of evolution through graphical analysis of various parameters.

Fig. 2 depicts the variation of ρ_p and $|\lambda|$ versus time in the decelerating and accelerating modes of Universe. It is observed that in the early Universe $\rho_p > |\lambda|$, and for sufficiently large time, ρ_p and $|\lambda|$ tend to zero. Therefore, the strings disappear from the Universe at late time (i. e. present epoch). According to Ref. [17], since there is no direct evidence of strings in the present-day Universe, we are in general, interested in constructing models of a Universe that evolves purely from the era dominated by either geometric string or massive strings and ends up in a particle dominated era with or without remnants of strings. Therefore, the above model describes the evolution of the Universe consistent with the present-day observations. $\frac{\rho_p}{|\lambda|}$ has been graphed in **Fig. 3**. It is evident that $\frac{\rho_p}{|\lambda|}$ is decreasing function of time and $\frac{\rho_p}{|\lambda|} > 1$ in the early Universe. This shows that massive strings dominate the early Universe. But the string tension density drops to zero for sufficiently large times. That is why the strings are not observed in current astronomical observations.

From equation (28), it is clear that displacement vector (β) is decreasing function of time and approaches to small positive value. This behaviour of β is clearly shown in **Fig. 4**, in accelerating and decelerating phase

of the Universe. From equation (34), it is observed that for $n < 3$ and $m = 0.5$, the density parameter (Ω) approaches to 1 at late time. Thus the derived model predicts a flat Universe at the present epoch. **Fig. 5** depicts the variation of density parameter versus time during the evolution of Universe.

3.2 Lyra's Cosmology of String fluid with Exponential-law

Solving the equations (17), (12) and (19), we obtain the metric functions as

$$A(t) = c_2^{\frac{3m}{m+1}} \exp\left(\frac{3m\ell}{m+1}t\right), \quad (41)$$

$$B(t) = c_4 c_2^{\frac{3m}{m+1}} \exp\left[\frac{3m\ell}{m+1}t + \frac{c_2^{\frac{6(m-1)}{m+1}}(m+1)^2}{18\ell^2(3m-1)(m-1)}e^{\frac{6\ell(m-1)}{m+1}t} - \frac{c_3}{3\ell c_2^3}e^{-3\ell t}\right], \quad (42)$$

$$C(t) = c_4^{-1} c_2^{\frac{3(1-m)}{m+1}} \exp\left[\frac{3(1-m)\ell}{m+1}t - \frac{c_2^{\frac{6(m-1)}{m+1}}(m+1)^2}{18\ell^2(3m-1)(m-1)}e^{\frac{6\ell(m-1)}{m+1}t} + \frac{c_3}{3\ell c_2^3}e^{-3\ell t}\right]. \quad (43)$$

Equation (16) leads to

$$\frac{\dot{\beta}}{\beta} = -3\ell \quad (44)$$

Integrating equation (35), we obtain

$$\beta = k_0 e^{-3\ell t} \quad (45)$$

where k_0 is an integrating constant.

The expressions for the isotropic pressure, the proper energy density, the string tension and the particle density for the derived model are obtained as

$$\begin{aligned} p = & -\frac{9\ell^2(m^2 - m + 1)}{(m+1)^2} + \frac{c_2^{\frac{6(m-1)}{m+1}}(m+1)}{4(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} - \frac{2c_3 c_2^{\frac{3(m-3)}{m+1}}(m+1)}{3\ell(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & + \frac{3c_3(1-2m)}{c_2^3(m+1)}e^{-3\ell t} - \frac{c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{9\ell^2(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} - \frac{c_3^2}{c_2^6}e^{-6\ell t} - \frac{3}{4}k_0^2 e^{-3\ell t}, \end{aligned} \quad (46)$$

$$\begin{aligned} \rho = & \frac{9\ell^2 m(2-m)}{(m+1)^2} + \frac{c_2^{\frac{6(m-1)}{m+1}}(5-11m)}{4(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} - \frac{2c_3 c_2^{\frac{3(m-3)}{m+1}}(m+1)}{3\ell(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & + \frac{3c_3(1-2m)}{c_2^3(m+1)}e^{-3\ell t} - \frac{c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{9\ell^2(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} - \frac{c_3^2}{c_2^6}e^{-6\ell t} + \frac{3}{4}k_0^2 e^{-3\ell t} \end{aligned} \quad (47)$$

$$\lambda = \frac{9\ell^2(2m-1)}{m+1} + 2c_2^{\frac{6(m-1)}{m+1}}e^{\frac{6\ell(m-1)}{m+1}t}, \quad (48)$$

$$\begin{aligned} \rho_p = & \frac{9\ell^2(1+m-3m^2)}{(m+1)^2} + \frac{c_2^{\frac{6(m-1)}{m+1}}(13-25m)}{4(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} - \frac{2c_3 c_2^{\frac{3(m-3)}{m+1}}(m+1)}{3\ell(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & + \frac{3c_3(1-2m)}{c_2^3(m+1)}e^{-3\ell t} - \frac{c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{9\ell^2(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} - \frac{c_3^2}{c_2^6}e^{-6\ell t} + \frac{3}{4}k_0^2 e^{-3\ell t}. \end{aligned} \quad (49)$$

The critical energy density (ρ_c) and density parameter (Ω) are given by

$$\rho_c = 3\ell^2 \quad (50)$$

$$\begin{aligned}\Omega = & \frac{3m(2-m)}{(m+1)^2} + \frac{c_2^{\frac{6(m-1)}{m+1}}(5-11m)}{12\ell^2(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} - \frac{2c_3c_2^{\frac{3(m-3)}{m+1}}(m+1)}{9\ell^2(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & + \frac{3c_3(1-2m)}{3\ell^2c_2^3(m+1)}e^{-3\ell t} - \frac{c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{27\ell^4(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} - \frac{c_3^2}{3\ell^2c_2^6}e^{-6\ell t} + \frac{1}{4\ell^2}k_0^2e^{-3\ell t}\end{aligned}\quad (51)$$

The average Hubble's parameter, expansion scalar, shear scalar spatial volume (V), and anisotropy parameter (\bar{A}) of the model are, respectively given by

$$H = \ell. \quad (52)$$

$$\theta = 3\ell, \quad (53)$$

$$\begin{aligned}\sigma^2 = & \frac{3\ell^2(2m-1)^2}{(m+1)^2} - \frac{c_2^{\frac{6(m-1)}{m+1}}(5-11m)}{4(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} + \frac{2c_3c_2^{\frac{3(m-3)}{m+1}}(m+1)}{3\ell(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & - \frac{3c_3(1-2m)}{c_2^3(m+1)}e^{-3\ell t} + \frac{c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{9\ell^2(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} + \frac{c_3^2}{c_2^6}e^{-6\ell t},\end{aligned}\quad (54)$$

$$V = c_2^3e^{3\ell t}, \quad (55)$$

$$\begin{aligned}\bar{A} = & \frac{2(2m-1)^2}{(m+1)^2} - \frac{c_2^{\frac{6(m-1)}{m+1}}(5-11m)}{6\ell^2(3m-1)}e^{\frac{6\ell(m-1)}{m+1}t} + \frac{4c_3c_2^{\frac{3(m-3)}{m+1}}(m+1)}{9\ell^3(3m-1)}e^{\frac{3\ell(m-3)}{m+1}t} \\ & - \frac{2c_3(1-2m)}{\ell^2c_2^3(m+1)}e^{-3\ell t} + \frac{2c_2^{\frac{12(m-1)}{m+1}}(m+1)^2}{27\ell^4(3m-1)^2}e^{\frac{12\ell(m-1)}{m+1}t} + \frac{2c_3^2}{3\ell^2c_2^6}e^{-6\ell t},\end{aligned}\quad (56)$$

The value of DP (q) is found to be

$$q = -1. \quad (57)$$

Recent observations of SN Ia [43]–[47] suggest that the Universe is accelerating in its present state of evolution. It is believed that the way Universe is accelerating presently; it will expand at the fastest possible rate in future and forever. For $n = 0$, we get $q = -1$; incidentally this value of DP leads to $dH/dt = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the Universe. Therefore, the derived model can be utilized to describe the dynamics of the late time evolution of the actual Universe. From equation (51), it is clear that the density parameter (Ω) approaches to 1 at late time. Thus the derived model predicts a flat Universe.

4 Concluding Remarks

In this paper, we have studied Bianchi-II string cosmological models in normal gauge for Lyra's manifold with constant deceleration parameter, considering two cases, (3.1) and (3.2). for $n \neq 0$ and $n = 0$ respectively. It is observed that in both cases, the mean anisotropy parameter (\bar{A}) drops to zero for $m = 0.5$ for sufficiently large times. Thus for $m = 0.5$, isotropy is achieved in derived models at late time. The main features of the work are as follows:

- The models are based on exact solutions of modified Einstein's field equations in normal gauge for Lyra's manifold for the anisotropic Bianchi-II space-time filled with massive strings.
- The singular model ($n \neq 0$) seems to describe the dynamics of Universe from big bang to the present epoch while the non-singular model ($n = 0$) seems reasonable to project dynamics of future Universe.
- The Universe acquires flatness at late time (see **Fig. 5**) which is consistent with the prediction of current observations.
- The displacement vector (β) is a decreasing function of time and it approaches to small positive value at late time. This matches with the nature of cosmological constant (Λ).

- The string tension density vanishes for sufficiently large times (see **Fig. 2**). Thus the strings disappear from Universe at late time. That is why, the strings are not observable in the present Universe.
- The age of Universe in singular model is given by

$$T_0 = \frac{1}{n}H_0^{-1} - \frac{c_1}{n\ell}$$

which differ from present estimation i. e. $T_0 = H_0^{-1} \approx 14Gyr$. But if we take, $n = 1$ and $c_1 = 0$, the model is in good agreement with present age of Universe.

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